Constructing Noise-Reducing Operators from Training Images

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Abstract

We discuss constructing non-linear noise reduction operators on binary images using a training set of noiseless images. We extract from the training set a probability distribution over local neighborhoods. Our operator changes pixel values when such a change turns a low probability neighborhood into high probability one.

Keywords and Phrases

non-linear operator; noise reduction; image reconstruction; edge relaxation; probabilistic relaxation

1 Motivation

We have developed an operator that modifies a pixel based on the probability of the current neighborhood configuration of that pixel. If the probability of a neighborhood configuration is below a given probability, $p*$, and a change will bring the probability above $p*$, then the neighborhood configuration is changed.

To implement this operator, we gather information about different neighborhood configurations by sampling neighborhoods from a training set of noiseless images (Milun and Sher, 1991). From this information, we generate a probability distribution of neighborhood configurations. There are several motivations for using this technique; it is quick, flexible and easy to implement.

The implementation is separated into three parts: data gathering, probability distribution construction, and noise reduction. No "complex" arithmetic is involved in any of these steps, and all calculations (additions and a few divisions) are performed off-line as part of the generation of the probability distribution. During the execution of the operator, we merely look up any probabilities we need and compare them to $p*$, eliminating floating point arithmetic, therefore providing us with quick response times.

The noise reduction is controlled both by the probability distribution and by the threshold, t, specified at run time, from which $p*$ is determined. The list of probabilities is dependent upon the "training images" used, and these training images can be changed to suit the requirements of the particular task.

2 Background

This section reviews Markov random field modelling and mathematical morphology. We demonstrate the relationship between previous work in these areas and our current line of research.

Markov Random Field Models

Most of the work on image processing that uses distributions over local neighborhoods falls under the rubric of Markov random fields (MRF). In this paper, we generate the probability distribution by sampling. Milun and Sher (Milun and Sher, 1991) developed this sampling technique for generating neighborhood distributions which they used for image reconstruction. They also extended this sampling to MRFs in noisy domains (double fields) (Milun and Sher, 1993; Milun and Sher, 1992).

Sher (Sher, 1991b) takes the alternative approach of generating the distribution by theoretically dropping objects of known shape into an image and calculating the probabilities of the various neighborhood configurations.

Gray, Kay and Titterington (Gray *et al.*, 1992) follows up on work by Derin and Elliott (Derin and Elliott, 1987) on the estimation of MRF parameters using a method called the "logit" method, and a "histogram count", which is very similar to the sampling ideas of our technique. However, their methods require that the noise be uncorrelated and do not make use of training image sets.

Lakshmanan and Derin's (Lakshmanan and Derin, 1989) goals are similar to those of Milun and Sher (Milun and Sher, 1991). They try for a data-driven approach, with almost no external parameters. However they still require the noise to be independent. They estimate the field from the image itself, using two interleaved simulated annealing processes, and do not derive the field from a training set. Manjunath and Chellappa's approach (Manjunath and Chellappa, 1991) is similar to Lakshmanan and Derin's, but they show that a nearest neighbor technique works almost as well as simulated annealing.

Markov random field models are of continuing interest as a basis for image analysis. While their main use has been in image reconstruction and segmentation, they can also be used to generate the probability distribution we are interested in. Various estimators have been used to consider a reconstructed field as a "good fit" to an MRF probability field, for example, MAP (maximum *a posteriori*) (Besag, 1986;

Geman and Geman, 1984; Geman *et al.*, 1990; Goutsias, 1991; Johnson *et al.*, 1991; Tan *et al.*, 1992; Bedini and Tonazzini, 1992; Lugosi, 1992), and MPM (most probable marginals) (Marroquin *et al.*, 1985; Sher, 1991a); and various methods of generating these estimates have been used, for example, simulated annealing (Geman and Geman, 1984; Bedini and Tonazzini, 1992; Blake, 1989; Metropolis *et al.*, 1953; Lakshmanan and Derin, 1989), ICM (Besag, 1986; Geman *et al.*, 1990; Karssemeijer, 1990) and refinements and modifications to parts of these methods (Swain, 1990; Sher, 1991a; Johnson *et al.*, 1991). However, not much attention has been paid to the methods of deriving the probability density functions.

Mathematical Morphology

Mathematical morphology is a new and exciting field created by Serra (Serra, 1982; Serra, 1986). Haralick, Sternberg and Zhuang (Haralick *et al.*, 1987a) discussed the relationship between image analysis and mathematical morphology.

Mathematical morphology is typically used to skeletonize objects. Thomas, Peters and Jeanty (Thomas *et al.*, 1991) used mathematical morphology, specifically skeletonization, to find the lengths of fetal ulnas. Jang and Chin (Jang and Chin, 1990) developed a set of morphological techniques and proved that their algorithms thin correctly.

Mathematical morphology can be used to perform edge detection, for example Lee, Haralick and Shapiro (S.J.Lee *et al.*, 1986). Klingler *et.al.* (Klingler Jr. *et al.*, 1988) used morphological operations to segment and smooth 2-D echocardiograms, while Revankar and Sher (Sher *et al.*, 1989) used mathematical morphology for adaptive thresholding on 2-D echocardiograms.

Mathematical morphology can also be applied to image reconstruction and pattern restoration. Given some smoothness criterion, Schonfeld (Schonfeld and Goutsias, 1991) found optimal morphological operators to trade off between smoothness and trueness to the original. Dougherty and Loce (Dougherty and Loce,

1992) presented a paradigm for library optimization and a methodology for first-order-library construction, which greatly reduces design computation while at the same time produce good filters

Mathematical morphologies also apply to the grayscale domain, with work done by Haralick, Sternberg and Zhuang (Haralick *et al.*, 1987b). Maragos (Maragos, 1989) proved theorems about threshold level invariances (such that doing gray-scale morphologies will result in the appropriate binary morphologies). Maragos also analyzed morphological operations and proved that the kernel of any morphological operation can be represented by a basis. Maragos used this idea to develop a fast implementation for small median filters.

Mathematical morphologies are powerful non-linear local techniques. Our operator is similar to mathematical morphologies in that respect; it is another non-linear local technique.

3 Implementation

Initially, we generate a probability distribution for local neighborhoods by sampling training images without noise from the desired domain. This distribution is then sorted and $p*$ is selected: above which we will "accept" an instance as likely and below which we will "reject" an instance as unlikely. Given threshold t, let $p* = p_n$ such that $\sum_{i=1}^{n} p_i \leq t$, and $\sum_{i=1}^{n+1} p_i > t$, where the p_i's are the probabilities sorted in decreasing order. Figure 1 contains some examples of 3×3 neighborhoods and the probability that they will appear in an image.

Gathering information on neighborhoods for a pixel in the "inside" of an image is simple; we look at its neighbors and keep the information in a table. For a pixel on the "boundary" of an image: the partial neighborhood surrounding a border pixel could have been from any of the "full" neighborhoods that contain this "partial" neighborhood; thus the probability of a "partial" neighborhood is the sum of the probability of the “full” neighborhoods that contain it. (See figure 2.)

Our operator randomly selects a pixel from the image and compares the probability of the neighborhood of this pixel (current probability) with the probability for a similar neighborhood which differs only in the center pixel (toggled probability). If the current probability is likely (above $p*$) then the pixel remains unchanged. Also, if the current and toggled probabilities are both unlikely (below $p*$), the pixel again remains unchanged. However, if the current probability is unlikely but the toggled probability is likely, then do we modify the center pixel. This process continues until the image no longer changes, that is, until all neighborhood configurations are “acceptable” configurations.

4 Results

Our method produces some very promising results. We gather information from 100 artificial scenes generated by randomly dropping rectangles and ellipses of various sizes and orientations. We construct our list of probabilities from these “training images,” and test our operator on images which are random rectangles and ellipses with noise. Figure 3 is an example of our results.

We can achieve rapid noise reduction for any domain using this technique by training with sample images from the desired domain, or from a similar domain. Figure 4 shows tests performed on an image made up of rectangles and ellipses that was designed to be a scene with a house. Since we are in a “similar” domain, our probabilities and operator did well in filtering out the noise.

We performed a study of the two primary parameters: the different noise levels (from 3% to 9%) and the threshold, t, used to filter out the noise (from 0.65 to 0.95). Figures 5, 6, and 7 are plots of the min-max-mean results of runs performed on ten different images. These graphs plot t against the number of incorrect bits remaining (after filtering). We studied thresholds from 0.55 to 0.95 at 0.05 intervals. From the initial data,

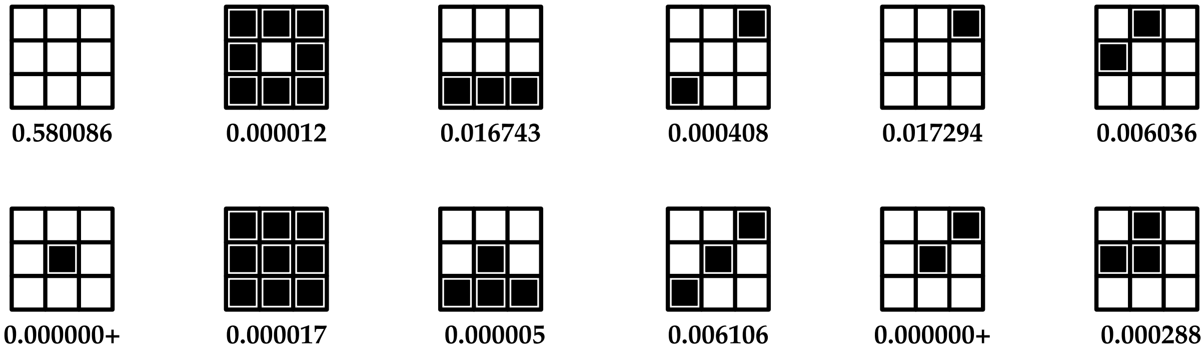

Figure 1: Sample neighborhoods and their associated probability.

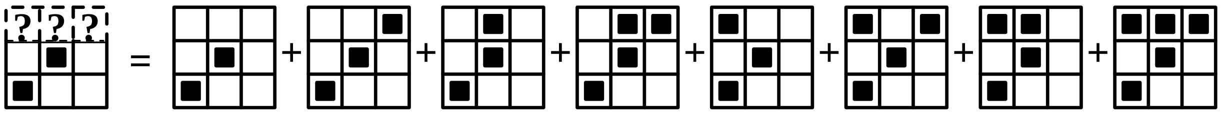

Figure 2: An example partial neighborhood (top border).

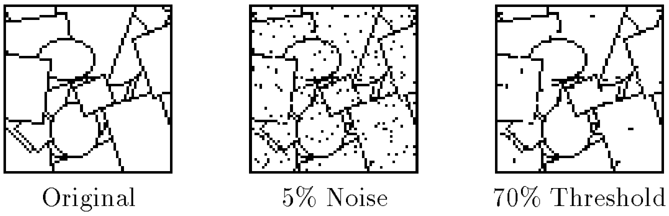

Figure 3: Random rectangles and ellipses.

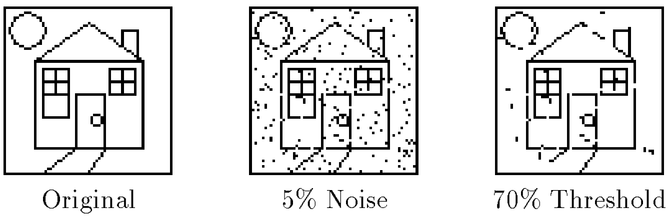

Figure 4: Scene drawn with rectangles and ellipses.

the minimum appeared to lie between 0.65 and 0.75; therefore we performed additional studies at thresholds from 0.65 to 0.75 at 0.01 intervals.

We generate plots similar to those previously mentioned for noise levels from 3% to 9%. However, in order to show the different noise levels and how they compare, we combine them in figure 8. Since images with high noise will have a higher number of incorrect bits remaining, the number of incorrect bits remaining is a misleading measure. Therefore, we use the ratio of remaining noise to initial noise ratio as a measure of comparison in this graph.

By inspection, we can determine that 0.75 is the optimal t for this domain (randomly placed rectangles and ellipses). A threshold in this range can be justified as follows: if t were too low, we would be allowing almost all neighborhood configurations; if t were too high, we would be doing the reverse, disallowing almost all neighborhood configurations.

5 Discussion

We developed a quick, easy, and flexible method for reducing noise in binary images. It uses a non-linear local operator with some global knowledge that lends itself to be parallelized. Some further investigations may involve looking at parallelizing this operator.

It may also be worth incorporating a noise model into our probability distribution by looking at the probabilities of noiseless neighborhoods that are changed to noisy configurations.

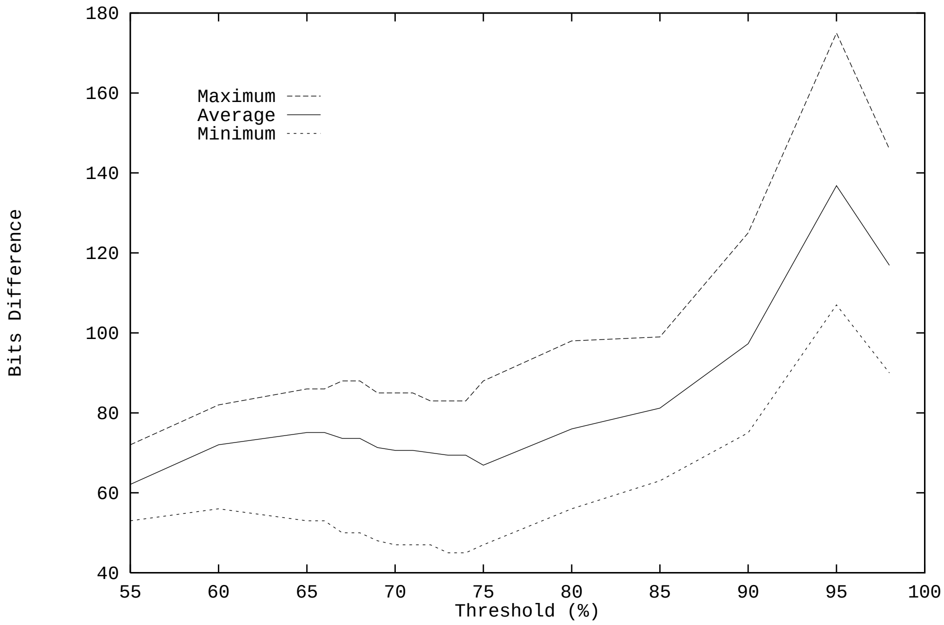

Figure 5: Threshold vs. Bits Difference for 3% Noise.

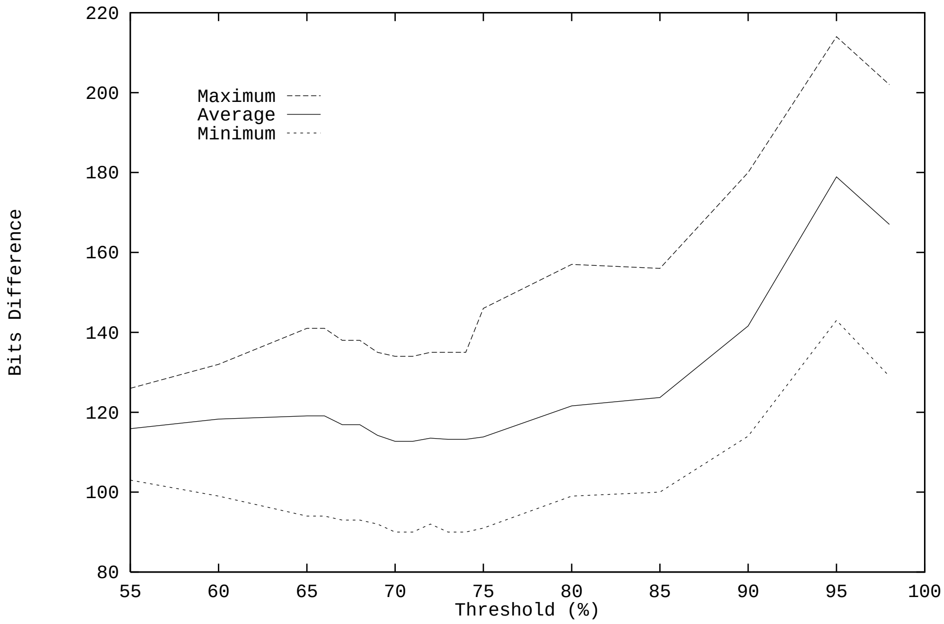

Figure 6: Threshold vs. Bits Difference for 5% Noise.

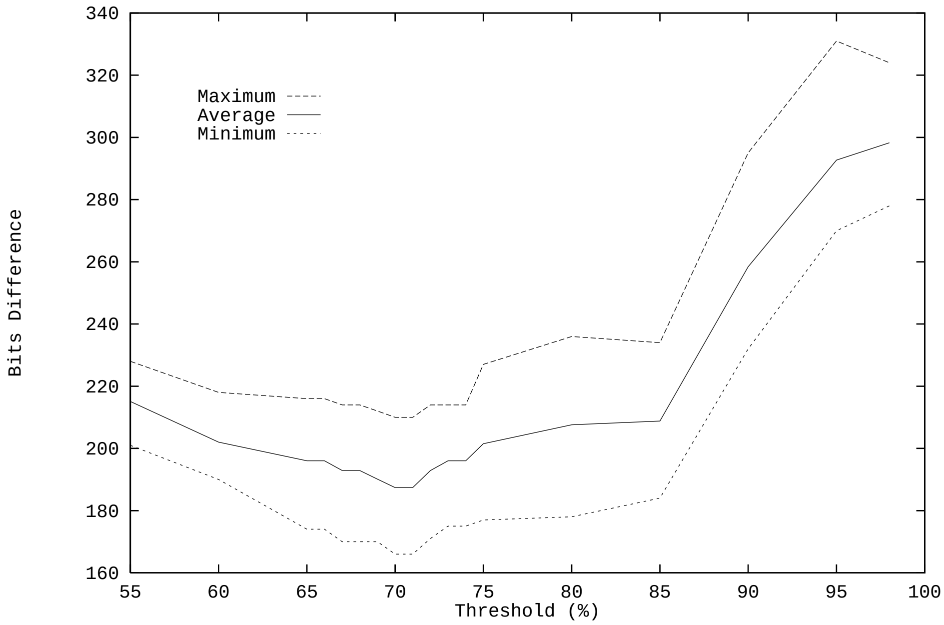

Figure 7: Threshold vs. Bits Difference for 8% Noise.

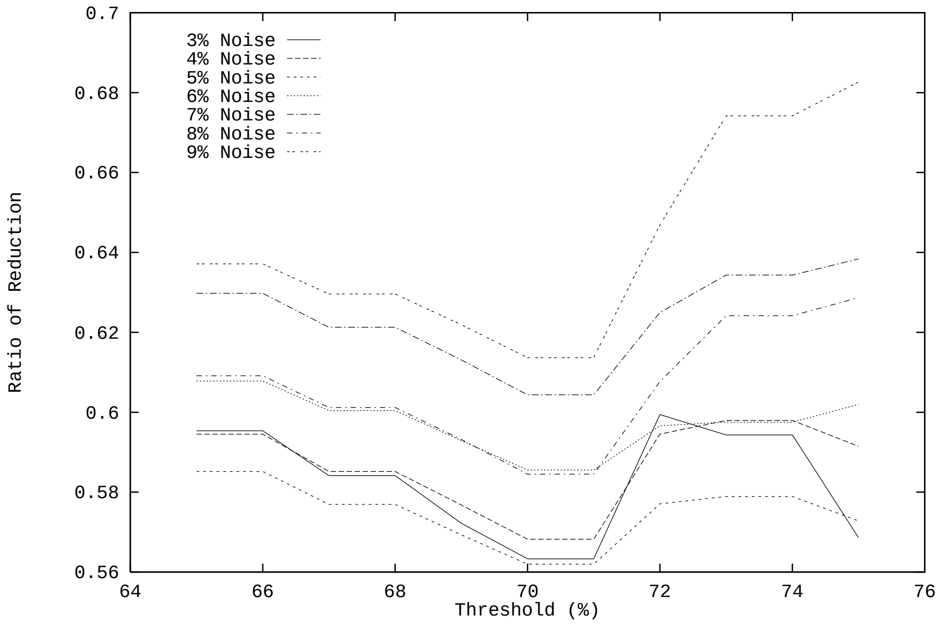

Figure 8: Thresholds for noise from 3% to 9%.

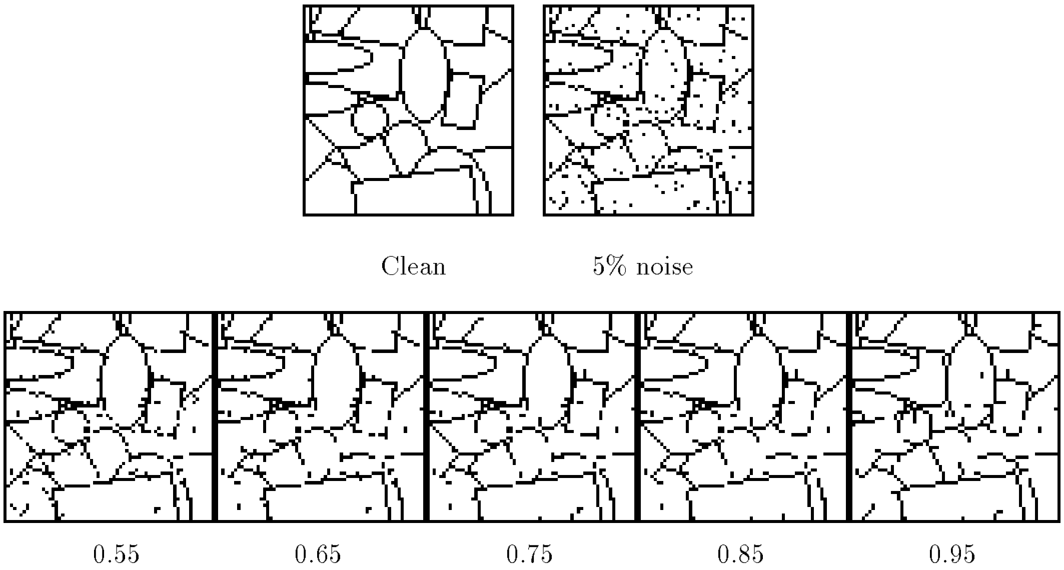

Figure 9: Original image. Image with 5% noise. Cleaned images from 0.55 to 0.95 thresholds with 0.05 increments.

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