

# On Fairness of Incentive-Compatible Multi-Radio Channel Assignment in Multiple Collision Domains

Tingting Chen    Sheng Zhong

Computer Science and Engineering Department, SUNY Buffalo

{tchen9, szhong}@buffalo.edu

## Abstract

Non-cooperative channel assignment, i.e., channel assignment for selfish wireless devices, is highly challenging, especially in multiple collision domains. In this paper, we study the problem of non-cooperative multi-radio channel assignment in multiple collision domains and focus on the fairness issue. We first conduct an analysis of the fairness property of the system assuming no incentive-compatible scheme is deployed. We show that, without any incentive-compatible scheme for channel assignment, the stable states of the system may well be max-min unfair. In order to guarantee fairness, we propose a channel assignment scheme for multiple collision domains that is incentive compatible. We rigorously show that our proposed scheme can always achieve NEs with complete fairness. Simulation results verify that our scheme guarantees complete fairness.

## I. INTRODUCTION

The problem of channel assignment has been studied extensively in wireless networks. In particular, given the increasing popularity of multi-radio wireless devices, a number of works have been done on the multi-radio channel assignment.

In many wireless networks, a lot of devices belong to selfish users who have their own interests. These selfish users will let their devices deviate from the protocols as long as this will benefit

themselves. Hence, it is important to design incentive-compatible channel assignment schemes that can function well in the presence of selfish users—this is called *non-cooperative* multi-radio channel assignment, because the involved devices are selfish rather than cooperative.

Existing works (e.g., [1]–[4]) have studied non-cooperative multi-radio channel assignment, but they are restricted to the setting of a single collision domain. Unlike these works, in this paper, we consider non-cooperative multi-radio channel assignment in multiple collision domains. Specifically, we focus on the fairness of channel assignment in such a setting.

Our study mainly consists of two parts. In the first part, we assume that there is no incentive-compatible scheme deployed, and analyze the fairness property of the stable states that the system will converge to. Our analysis is in a game theoretic model, which allows us to obtain quantified results on the fairness of the stable states (namely *Nash Equilibria* (NEs) in the game model). The results indicate that the system could reach some NEs that are max-min unfair.

To guarantee fairness, we propose an incentive-compatible scheme for channel assignment in multiple collision domains. As in many existing works on non-cooperative wireless networks (e.g., [1], [4]–[6]), the main tool used to provide incentives is payment (of credit, or virtual money). Note that it is reasonable to require users to pay for the channels they are assigned to, because communication channels are a type of scarce resources.

The following is a summary of our contributions in this paper:

- This is the *first* work to study fairness of non-cooperative multi-radio channel assignment in multiple collision domains.
- Assuming there is no incentive compatible scheme, we analyze the fairness property of NEs in the game of multi-radio channel assignment in multiple collision domains. We find that there are cases in which most NEs are max-min unfair.
- We propose an incentive-compatible scheme for multi-radio channel assignment in multiple collision domains. Through rigorous analysis, we show that our scheme provides complete fairness to all nodes.
- Simulations in GloMoSim [7] verify that our scheme guarantees complete fairness.

The rest of this paper is organized as follows. In Section II we present technical preliminaries. In Section III we analyze the fairness property assuming no incentive-compatible channel assignment scheme is used. In Section IV, we propose our incentive-compatible scheme for multi-radio channel assignment. Evaluation results are presented in Section V. We conclude our

paper in Section VI.

## II. TECHNICAL PRELIMINARIES

In this section, we describe the system model in which we analyze non-cooperative channel assignment. Specifically, we define a game of channel assignment in multiple collision domains. We also review some game theoretic definitions used in later parts of this paper.

### A. Model

As in [1], [2], [4], we assume that the available frequency band is divided into orthogonal channels. Denote the set of channels by  $C$ . In the system, each node has  $K$  transceivers and can establish a bidirectional communication with another node, by tuning a pair of transceivers (one transceiver from each node) to the same channel. Each node can transmit packets to another node using multiple channels. Denote by  $P$  the set of communicating node pairs. There are  $|P|$  communicating pairs in the system and each node is only involved in one such node pair. We also assume that the channels have same characteristics. Note that there exist some node pairs that cannot interfere with the communications of some other pairs, even if they are all using the same channel. In other words, the communicating pairs are in multiple collision domains. We use the interference model (e.g. in [8]) that if two communicating pairs within each other's interference range are transmitting packets on the same channel at the same time, neither of them can successfully transmit any useful data.

Our model for multi-radio channel assignment in multiple collision domains is a strategic game. In the game, each player is a pair of communicating nodes. As in many existing works on channel assignment in multiple collision domains (e.g., [9]–[11]), we assume that time is divided into  $T$  slots, each of a fixed length, where  $T$  is a large number. In the channel assignment game, a player's strategy is his choice of channels for all time slots.

Formally, the strategy of player  $i$ ,  $a_i$  is defined as  $a_i = \{A_{i,c,t} | 1 \leq t \leq T, c \in C\}$ , where

$$A_{i,c,t} = \begin{cases} 1 & \text{if } i \text{ is using channel } c \text{ in slot } t, \\ 0 & \text{if } i \text{ is not using channel } c \text{ in slot } t. \end{cases}$$

The strategy profile  $a$  is a matrix that includes all players' strategies, i.e.,  $a = (a_1, a_2, \dots, a_{|P|})$ . Denote by  $a_{-i}$  the profile of all players except  $i$ .

A flow contention graph can represent the interference relationship among players. Define  $N_i$  to be the set of players who are connected with  $i$  in the flow contention graph. We call  $N_i$  the interference set of player  $i$ . Let  $n_{max} = \max_{i \in P} |N_i|$ .

Denote by  $k_{i,t}$  the number of channels used by player  $i$  in any time slot  $t$ , i.e.,  $\forall i, \forall t, k_{i,t} = \sum_{c \in C} A_{i,c,t}$ . To define the payoff function for each player  $i$ , we must note that in each time slot, whether it can successfully transmit packets depends on its strategy as well as those of players in its interference set. Assume that  $r$  is the amount of utility that a player can obtain by transmitting data through one radio in a time slot;  $\beta$  ( $\beta < r$ ) is the cost in each time slot for one radio to transmit data. Then, the payoff for each player  $i$  is defined in Eq. (1).

$$u_i = \sum_{t=1}^T \left( r \sum_{c \in C} (A_{i,c,t} \cdot \prod_{j \in N_i} (1 - A_{j,c,t})) - \beta \cdot k_{i,t} \right) \quad (1)$$

Note that any player  $i$  can successfully transmit on one channel only if no player in his interference set uses that channel. Eq. (1) reflects this fact by multiplying  $A_{j,c,t}$  by  $\prod_{j \in N_i} (1 - A_{j,c,t})$ .

### B. Definitions

Before analyzing the channel assignment game, we first review some of the definitions that we use in later parts of this paper.

*Definition 1:* (Nash Equilibrium (NE)) [12] The strategy profile  $a^* = \{a_1^*, a_2^*, \dots, a_{|P|}^*\}$  is a Nash equilibrium (NE) if for every player, we have that  $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$  for all strategy  $a_i$ .

The main objective of this paper is to study fairness in the channel assignment game described above. Here we distinguish two levels of fairness: max-min fairness [13], which is weaker, and complete fairness, which is stronger.

*Definition 2:* (Max-Min Fairness) A strategy profile  $a^{mmf}$  is max-min fair if for every strategy profile  $s$  such that there exists player  $i \in P$ ,  $u_i(a^{mmf}) < u_i(s)$ , there must exist another player  $j \in P$ ,  $u_j(s) < u_j(a^{mmf}) < u_i(a^{mmf})$ . Otherwise, it is max-min unfair.

*Definition 3:* (Complete Fairness) The strategy profile  $a^{cf}$  achieves complete fairness if the payoffs of all players are equal.

## III. ANALYSIS OF FAINESS WITHOUT INCENTIVE-COMPATIBLE SCHEMES

In this section, we rigorously analyze the NEs in a system without incentive-compatible schemes, studying their fairness. We first prove that, in some scenarios, some NEs are max-

min *unfair* to players in terms of payoffs. Then we give an concrete example showing that in some cases, most NEs are max-min unfair.

*Proposition 1:* With  $|C| \leq K$ , if a Nash equilibrium strategy profile  $a^*$  has an outcome s.t. for a clique  $E$  of size  $n$  in the flow contention graph,  $\exists i \in E, s.t. \forall j \in E, j \neq i, u_i^* < u_j^*$  and  $d(i) < 2n - 1$  (where  $d(i)$  is the degree of  $i$  in the flow contention graph), then  $a^*$  is not max-min fair.

*Proof:* First we show that in a Nash equilibrium  $a^*$ , the payoff of player  $i$  can be written as  $u_i^* = \sum_{0 < t \leq T} \sum_{c \in C} A_{i,c,t}^* (r - \beta)$ . Actually, if  $a^*$  is a NE, then  $\forall i, t, c$ , if  $A_{i,c,t}^* = 1$ ,  $\prod_{j \in N_i} (1 - A_{j,c,t}^*) = 1$ . To show this, suppose  $\exists i, t, c$  s.t.  $A_{i,c,t}^* = 1$  and  $\prod_{j \in N_i} (1 - A_{j,c,t}^*) = 0$ . We consider another strategy for  $i$ ,  $a'_i$ , which equals  $a^*$  except for  $A_{i,c,t}' = 0$ . Then we compare the utilities of player  $i$  taking strategy  $a_i^*$  and  $a'_i$  when the strategies of players remain the same.

$$\begin{aligned} u'_i - u_i^* &= A_{i,c,t}' \prod_{j \in N_i} (1 - A_{j,c,t}^*) r - \beta A_{i,c,t}' - (A_{i,c,t}^* \prod_{j \in N_i} (1 - A_{j,c,t}^*) r - \beta A_{i,c,t}^*) \\ &= 0 - (-\beta) > 0 \end{aligned}$$

This contradicts with the fact that  $a^*$  is a Nash equilibrium.

Now we suppose we have a clique  $A$  of size  $n$  in the flow contention graph. We denote the player with the strict minimum payoff in  $E$  by  $i$ , i.e.  $\forall j \in E, j \neq i, u_i^* < u_j^*$ .

We consider the two cases of  $d(i)$  to show that no matter in which case, player  $i$  can always increase its payoff without affecting the players with lower payoffs than  $i$ .

Case 1.  $d(i) = n - 1$ . For  $u_j^* > u_i^*$ ,  $\exists(t, c)$  s.t.  $A_{j,c,t}^* = 1, A_{i,c,t}^* = 0$ . In this time slot, all the other players do not have radios on channel  $c$ , i.e.  $\forall h \in E, h \neq j, A_{h,c,t}^* = 0$ , because otherwise  $h$  and  $j$  are interfering with each other and thus  $a^*$  would not be a Nash equilibrium. Since  $d(i) = n - 1$  means that the players in  $i$ 's interference set are all in  $E$ , by making  $S_{i,c,t} = 1$  and  $A_{j,c,t} = 0$ ,  $i$  can increase its payoff without decreasing others' payoffs except  $u_j^*$ . Therefore,  $a^*$  is not max-min fair.

Case 2.  $d(i) > n - 1$ . In this case, there are some players in  $i$ 's interference set but not in clique  $A$ . We denote the player set  $Q = \{q | q \in N_i, q \notin E, u_q \leq u_i\}$ . From  $|Q| + n - 1 \leq d(i) < 2n - 1$ , we can obtain that  $|Q| \leq n - 1$ .

Because  $\forall q \in Q, u_q^* \leq u_i^*$  and  $|Q| \leq n - 1$ , we get

$$(n - 1) \sum_{0 < t \leq T} \sum_{c \in C} A_{i,c,t}^* \geq \sum_{q \in Q} \sum_{0 < t \leq T} \sum_{c \in C} A_{q,c,t}^* \quad (2)$$

Furthermore, from  $u_j^* > u_i^*$ , we have

$$\sum_{0 < t \leq T} \sum_{c \in C} A_{j,c,t}^* > \sum_{0 < t \leq T} \sum_{c \in C} A_{i,c,t}^* \quad (3)$$

Combining (2) and (3),

$$\sum_{j \in E, j \neq i} \sum_{0 < t \leq T} \sum_{c \in C} A_{j,c,t}^* > (n-1) \sum_{0 < t \leq T} \sum_{c \in C} A_{i,c,t}^* \geq \sum_{q \in Q} \sum_{0 < t \leq T} \sum_{c \in C} A_{q,c,t}^* \quad (4)$$

Since  $\forall t, c, \sum_{j \in E, j \neq i} A_{j,c,t}^* \leq 1$ , to satisfy (4), there must exist  $t, c$ , s.t.  $\sum_{j \in E, j \neq i} A_{j,c,t}^* = 1$  and  $\sum_{q \in Q} A_{q,c,t}^* = 0$ . Then player  $i$  can increase its payoff by putting one spare radio on channel  $c$  in time slot  $t$ , while the payoffs of players in  $Q$  will not be decreased since no interference is caused with them.

This completes the proof of Proposition 1. ■

Using Proposition 1, now we demonstrate that, in some scenarios, most NEs are max-min *unfair*.

**Example.** Consider, the flow contention graph in Fig. 1.

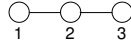


Fig. 1. The flow contention graph in example of unfairness.

In this example, we assume that each player has 2 radios and that there are 2 channels,  $a, b$ , available. It is easy to get that the Nash equilibrium channel assignment should fall in one of the cases shown in Fig. 2. Since player 1 and 2 form a clique of size 2 in the flow contention graph, by Proposition 1, a NE being max-min fair requires that  $u_1 = u_2$ . Similarly,  $u_2 = u_3$  is also required. This means that cases (I) and (III) need to happen in the same number of time slots. So, if the total number of slots  $T = 2$ , the number of max-min fair Nash equilibria is only 6 (4 NEs when neither (I) or (III) ever happens and 2 NEs when both (I) and (III) only happen in one time slot), while there are  $4^2 = 16$  Nash equilibria in total. Therefore, 62.5% of the Nash equilibria in this scenario are max-min unfair.

#### IV. SCHEME FOR COMPLETE FAIRNESS TO PLAYERS (SCF)

In this section, we provide an incentive-compatible channel assignment scheme to make sure that all the NEs provide complete fairness to all the players.

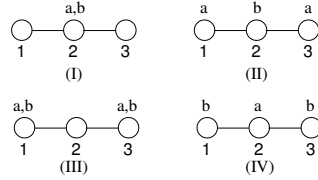


Fig. 2. Nash equilibrium channel assignment cases in any time slot. (The 'a' or 'b' denotes the channel(s) that the player assigns its radios to.)

As in many existing works on non-cooperative wireless networks (e.g., [1], [4]–[6], among others), we use an economic tool, payment, in our scheme. Assume that there is virtual currency circulating in the system. Before using channels, each player has to first pay the administrator for his use of the channels.

To calculate the payment for each player, we use the topological information of each node. Specifically, we assume that the flow contention graph is partitioned into a number of maximal independent sets, and that, before the channel assignment game starts, each player receives the ID of its independent set ID as an input. In practice, this independent set ID can be obtained in various ways. One possibility is that a central agency who knows the global topology runs an algorithm to find maximum independent sets in rounds. If such a centralized approach is not applicable or undesirable, we can also use a distributed algorithm with reasonable complexity, running locally at each player to find which independent set each player belongs to. To provide good performance as well as time efficiency, the idea of some approximation algorithms (e.g. [14]) for maximal independent set partition can be useful. However, we do not discuss the details of the algorithm for finding the independent set IDs, because it is orthogonal to the main problems we solve. In this paper, we assume that independent sets are sorted such that a smaller independent set ID implies that the independent set is larger.<sup>1</sup>

We outline our scheme, the Scheme for Complete Fairness (SCF), in Protocol 1. In this scheme, we use the maximal independent set IDs  $i.MISID$  to compute the amount of payment each player should make in each slot. Let  $m$  denote the total number of independent sets in the system. To achieve complete fairness, we need to give equal opportunities to all players for

<sup>1</sup>We also assume that after the maximal independent set partition, the number of maximal independent sets is greater than  $\left\lceil \frac{|C|}{K} \right\rceil$ , because otherwise it is trivial to have all radios assigned without any interference.

use of channels. We introduce a special independent set called the token independent set for each slot  $t$ , whose ID is denoted by  $\chi_\tau^t$ . Our main idea here is to let each independent set be the token independent set in a round-robin fashion. In each time slot, only the players in the independent sets with IDs smaller than  $\chi_\tau^t$  by no more than  $\lfloor \frac{|C|}{K} \rfloor$ , are encouraged to put their radios on channels to transmit packets (as discussed below). As each independent set is taking turns to be the token independent set, players can obtain complete fairness.

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**Protocol 1** Outline of Scheme for Complete Fairness (SCF)

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In time slot  $t$ ,

(a) Each player  $i$  computes

$$\chi_\tau^t = (t \bmod m).$$

$$\chi_i^t = (\chi_\tau^t - i_{MISID}) \bmod m.$$

(b) Each player  $i$  decides to use  $k_{i,t}$  radios and makes payment

$$p_{i,t} = \frac{(r - \beta) \cdot k_{i,t} \cdot \chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon}. \quad (5)$$

(c) If  $\chi_i^t < \lfloor \frac{|C|}{K} \rfloor$ ,  $i$  keeps changing its strategy until it gets all its radios transmitting successfully; If  $\chi_i^t = \lfloor \frac{|C|}{K} \rfloor$ ,  $i$  keeps changing its strategy until it gets  $|C| \bmod K$  radios transmitting successfully.

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When we plug in the payment formula Eq. (5) in Protocol 1 into the payoff function for each player introduced in Section II, we can obtain the following updated payoff function, assuming no collisions.

$$\begin{aligned} u_i &= \sum_{t=1}^T \sum_{c \in C} r \cdot (A_{i,c,t} \cdot \prod_{j \in N_i} (1 - A_{j,c,t})) - \beta \sum_{t=1}^T k_{i,t} - \sum_{t=1}^T \frac{(r - \beta) \cdot k_{i,t} \cdot \chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} \\ &= \sum_{t=1}^T r \cdot k_{i,t} - \beta \sum_{t=1}^T k_{i,t} - \sum_{t=1}^T \frac{(r - \beta) \cdot k_{i,t} \cdot \chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} \\ &= \sum_{t=1}^T (r - \beta) \left( 1 - \frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} \right) k_{i,t}. \end{aligned}$$

For each player  $i$ , if  $\chi_i^t$  is smaller than or equal to  $\lfloor \frac{|C|}{K} \rfloor$ , it will get higher payoff if it increases its number of radios to transmit packets (because  $\lfloor \frac{|C|}{K} \rfloor$  and  $\chi_i^t$  are both integers and we define  $0 < \epsilon < 1$ ). On the other hand, if  $\chi_i^t > \lfloor \frac{|C|}{K} \rfloor$ , the payoff will decrease as the player  $i$  uses more radios to transmit packets.

Formally, we have the following theorems. Theorem 1 shows that if our scheme is used,



the system will converge to NEs with certain properties. Theorem 2 says with these properties, complete fairness is guaranteed.

*Theorem 1:* If SCF is used, all the NEs satisfy that for all player  $i$ , in all time slot  $t$ ,

$$k_{i,t} = \begin{cases} K & \text{if } \chi_i^t < \lfloor \frac{|C|}{K} \rfloor \\ |C| \bmod K & \text{if } \chi_i^t = \lfloor \frac{|C|}{K} \rfloor \\ 0 & \text{o.w.} \end{cases} \quad (6)$$

*Proof:* First we show that the state  $a^*$  which satisfies (6) is a Nash equilibrium. Then we show that SCF will only reach the state  $a^*$ .

Now we show that state  $a^*$  which satisfies (6) is a Nash equilibrium, which guarantees that players do not have incentives to deviate from  $a^*$  unilaterally.

Let  $u_i'$  denote the payoff of  $i$  by taking other strategy  $a_i'$  that does not satisfy (6).  $k_{i,t}'$  is used in  $a_i'$ . Since interference will result in even lower payoffs for players, in the proof, we only consider the best payoffs possible by  $a_i'$ , i.e., assuming that there is no interference by  $a_i'$ .

There are 3 possible cases.

Case 1.  $\chi_i^t > \lfloor \frac{|C|}{K} \rfloor$ .  $u_i' - u_i^* = \sum_{t=1}^T (r - \beta) \left( 1 - \frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} \right) k_{i,t} \leq 0$ , because  $\chi_i^t$  and  $\lfloor \frac{|C|}{K} \rfloor$  are integers and thus  $\frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} > 1$ .

Case 2.  $\chi_i^t < \lfloor \frac{|C|}{K} \rfloor$ .  $u_i' - u_i^* = \sum_{t=1}^T (r - \beta) \left( 1 - \frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} \right) (k_{i,t}' - K) \leq 0$ , since  $\frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} < 1$  and  $k_{i,t}' - K \leq 0$ .

Case 3.  $\chi_i^t = \lfloor \frac{|C|}{K} \rfloor$ . If other players not in Case 3 follow the channel assignment results as in (6), the number of channels that player  $i$  can use without interference is at most  $|C| \bmod K$  (i.e.  $k_{i,t}' \leq |C| \bmod K$ ). Also from  $\frac{\chi_i^t}{\lfloor \frac{|C|}{K} \rfloor + \epsilon} < 1$ . We can obtain that  $u_i'(a_i', a_{-i}^*) - u_i^*(a_i^*, a_{-i}^*) \leq 0$ .

To show that SCF will always reach the state that satisfies (6), we first notice that (6) is achievable within the system capacity. By Eq. (6), only players in  $\lfloor \frac{|C|}{K} \rfloor - 1$  independent sets use  $K$  radios and the player in 1 the independent set of size  $\lfloor \frac{|C|}{K} \rfloor$  use  $|C| \bmod K$  radios. When players in the same independent set allocate their radios on the same set of channels, the total number of channels without interference required by (6) is  $(\lfloor \frac{|C|}{K} \rfloor - 1)K + |C| \bmod K$ , which is no more than the number of channels in the system  $|C|$ . On the other hand, the system will not stabilize in any state that does not satisfy (6), due to the strategy changing conditions in SCF (step (c)). Hence the probability of reaching the state satisfying (6) from any state is positive.

Therefore, SCF always converges to NEs which satisfy (6). ■

*Theorem 2:* (Complete Fairness) Suppose that  $T \gg m$ . If SCF is used, every NE is completely fair.

*Proof:* First we note that in any  $m$  continuous timeslots, each independent set ID in  $\{1, 2, \dots, m\}$  happens to equal  $\chi_\tau^t$  once, due to the definition of  $\chi_\tau^t$ . So from (6), in any NE, for each player  $i$ , there is one time slot in which he uses  $|C| \bmod K$  radios. Also it is not difficult to get that for each player  $i$ , he fully utilizes  $K$  radios in  $\lfloor \frac{|C|}{K} \rfloor$  time slots. The payoff of each player in any  $m$  continuous timeslots is  $(r - \beta)(K \cdot \lfloor \frac{|C|}{K} \rfloor + |C| \bmod K)$ . Given that  $T \gg m$ ,  $\frac{T}{m}$  is an integer. Therefore, the payoff of each player in the entire game is  $\frac{T}{m}(r - \beta)(K \cdot \lfloor \frac{|C|}{K} \rfloor + |C| \bmod K)$ , which implies complete fairness. ■

## V. EVALUATIONS

In this section, we conduct simulations in GloMoSim [7] to study SCF. Maximal independent sets are computed before the start of the game by the approximation algorithm in [14].

The simulations are done in a randomly generated network of 20 pairs of nodes, as illustrated in Fig. 3 (where each dot represents a pair). Each pair consists of two nodes 20 meters apart. Within each pair, the data flow is bidirectional at a constant rate. The length of each time slot is set to be 30 seconds.

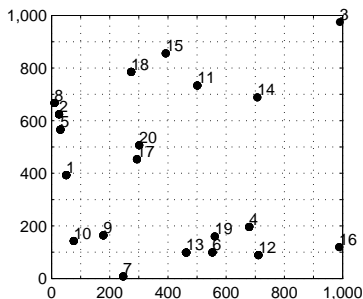


Fig. 3. The flow contention graph of the network.

We test our scheme SCF in two settings. In one setting, we let the number of channels  $|C| = 12$ , channel capacity  $R = 54$  Mbps, and the number of radios  $K = 4$ . In the other setting, we let  $|C| = 3$ ,  $R = 11$  Mbps and  $K = 2$ . We take the average of results over 500 runs.

In Section V-A, we evaluate SCF by measuring the fairness of the stable states of the system, and compare it with the fairness of NEs when there is no incentive-compatible schemes. In

Section V-B we measure the fairness in the processes of system convergence.

### A. Evaluation of Fairness

To measure the fairness among players, we utilize Jain’s fairness index [15] as a quantitative metric. Fairness index is a real number, ranging from 0.05(worst) to 1(best) for the system of 20 players. We measure the fairness indices of the system’s stable states achieved by SCF and the average fairness indices of random NEs, which should be reached when there is no incentive-compatible channel assignment scheme. We repeat the experiments with different traffic rates . The results are shown in Fig. 4 and 5, where the curves for “SCF” represent the fairness indices for SCF, and the curves for “No Protocol” represent the fairness indices when there is no incentive compatible scheme. We can see that for both settings ( $|C| = 3, K = 2$  and  $|C| = 12, K = 4$ , respectively), SCF guarantees that the system has fairness indices very close to 1, or even equal to 1. This verifies the effectiveness of SCF in achieving complete fairness, which is much better than the average fairness indices of NEs when there is no incentive compatible scheme.

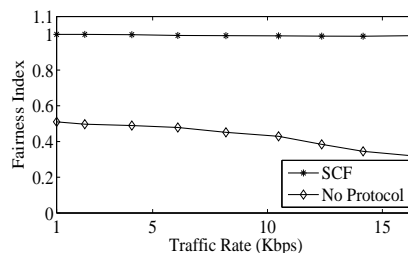


Fig. 4. Fairness index. ( $|C| = 3, K = 2, R = 11Mbps.$ )

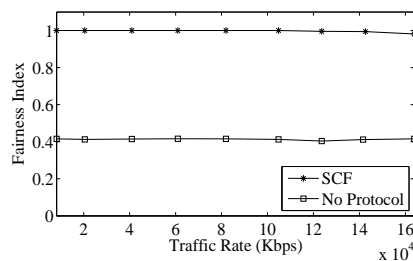


Fig. 5. Fairness index. ( $|C| = 12, K = 4, R = 54Mbps.$ )

### B. System Convergence

We are not only interested in fairness of the stable states, but also interested in fairness of the dynamic convergence process. In this subsection, we examine fairness of the processes the system converges to a stable state. We keep track of the fairness index value for SCF when the systems are converging to the stable state, and show the results in Fig. 6. In this experiment, the traffic demand rate is 80 Mbps. We can see that, within about 1000 seconds, the fairness index gets close to 1.

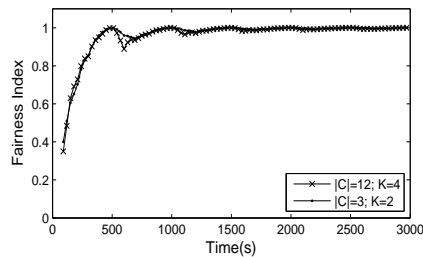


Fig. 6. Convergence of fairness index of SCF.

## VI. CONCLUSION

In this paper, we study the fairness of non-cooperative multi-radio channel assignment in multiple collision domains, and obtain two major results. The first major result is that, without an incentive-compatible channel assignment scheme, the system is likely to converge to NEs that are max-min unfair to the players. The second major result is an incentive-compatible scheme we design for multi-radio channel assignment in multiple collision domains, and a formal proof that the scheme guarantees complete fairness. Experiments results have verified the results of theoretical analysis.

## REFERENCES

- [1] Tingting Chen, S. Zhong. Perfectly Fair Channel Assignment in Non-cooperative Multi-radio Multi-channel Wireless Networks, *Computer Communications*, Volume 32, Number 6, Pages 1058-1061, 2009.
- [2] M. Felegyhazi, M. Cagalj, S. S. Bidokhti, and J.-P. Hubaux, *Noncooperative multi-radio channel allocation in wireless networks*, in *IEEE INFOCOM'07*, May 2007.
- [3] L. Gao and X. Wang, *A Game Approach for Multi-Channel Allocation in Multi-Hop Wireless Networks*, in *ACM MOBIHOC'08*, May 2008.

- [4] F. Wu, S. Zhong and C. Qiao, *Globally Optimal Channel Assignment for Non-cooperative Wireless Networks*, in IEEE INFOCOM'08, May 2008.
- [5] W. Wang, X. Y. Li, and Y. Wang, *Truthful multicast in selfish wireless networks*, in ACM MobiCom'04, Sep. 2004.
- [6] W. Wang, S. Eidenbenz, Y. Wang, and X. Y. Li, *Ours: Optimal unicast routing systems in non-cooperative wireless networks*, in ACM MobiCom'06, Sep. 2006.
- [7] UCLA Parallel Computing Laboratory. GloMoSim - Global Mobile Information Systems Simulation Library. Webpage, February 2006. <http://pcl.cs.ucla.edu/projects/glomosim/>.
- [8] Z. Fang and B. Bensaou. *Fair Bandwidth Sharing Algorithms based on Game Theory Frameworks for Wireless Ad-hoc Networks*, in IEEE INFOCOM'04, May, 2004.
- [9] X. Qin and R. Berry, *Exploiting multiuser diversity for medium access control in wireless networks*, in IEEE INFOCOM, 2003.
- [10] X. Lin, S. Rasool, A Distributed Joint Channel-Assignment, Scheduling and Routing Algorithm for Multi-Channel Ad-hoc Wireless Networks, in Proc. of IEEE INFOCOM 2007.
- [11] X. Li, A. Nusairat, Y. Wu, Y. Qi, J. Zhao, X. Chu, Y. Liu, *Joint Throughput Optimization for Wireless Mesh Networks*, IEEE Transactions on Mobile Computing, vol. 8, no. 7, pp. 895-909, July 2009, doi:10.1109/TMC.2008.160.
- [12] M. J. Osborne and A. Rubenstein, *A Course in Game Theory*. The MIT Press, 1994.
- [13] D. Bertsekas and R. Gallager. *Data Networks*. Prentice-Hall, 2nd ed., 1992.
- [14] M. M. Halldorsson and J. Radhakrishnan, *Greed is good: Approximating independent sets in sparse and bounded-degree graphs*, ACM STOC'94, 1994.
- [15] R. Jain, D.M. Chiu, and W. Hawe. *A Quantitative Measure of Fairness and Discrimination for Resource Allocation in Shared Systems*. DEC Research Report TR-301, 1984.