Bounding the Energy Consumption of Mobile Sensor Nodes For Triangulation-based Coverage

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***Abstract*— In a Triangulation-based coverage scheme, a group of three mobile sensor nodes (MSN) position themselves to form an equilateral triangle for a desired period of time. Such a scheme has several applications in localization, environmental monitoring and coordinated target tracking. In this work, we introduce an efficient mobile traversal algorithm (MTA) that profides a triangulation-based coverage of a field that can be approximated as a rectangle. We analyze the energy consumption of the MTA interms of the distance and time taken to complete the full coverage of the field. The bounds on the minimum total and individual energy consumption per MSN is determined. A prior knowledge about the energy consumption can be useful to charge the mobile seeds with the required amount needed for the particular application.**

I. Introduction

A fundamental issue in sensor network is the *coverage problem*, which addresses how well sensors observe or monitor a region. Due to the energy constraints of sensor nodes, an energy-efficient coverage scheme is viable for the lifetime of the sensor network. Several work exists in the literature on an energy-efficient coverage of a sensor field [1]–[4]. In [2]–[4], energy-efficiency is achieved through selective activation/deactivation of sensor nodes with an optimum scheduling procedure. In addition to scheduling the sensor's sleep and awake period, the sensing range of the sensors is also adjusted to conserve power in [1]. A reduction in the transmitted data achieves an energy-efficent target tracking scheme in [5].

In most of the prior works, the sensor nodes are stationary, which restricts the application of such schemes. We believe a mobile triangulation-based coverage scheme, in addition to monitoring a region, can support various others applications like localization, search operation and coordinated target tracking. For instance, in localization, angular positions or bearing of the MSNs are used to determine the location of the sensed nodes [6], [7]. In target tracking [8], MSNs move in a triangular formation to maintain coordination among themselves.

In this work, we propose a mobile traversal algorithm (MTA) that employs three MSNs, equipped with location devices such as GPS, to cover a field that can be approximated by a rectangle. The MSNs can be deployed at a random initial position in the field, and with the knowledge of their current location with respect to the field to be covered, they will form an equilateral triangle near the deployment site. Each side of the triangle is assumed to be r units, where r is an application dependant parameter, often limited by the sensing and communication ranges of the MSNs. After forming such a triangle for a pre-determined (constant) period of time, they will coordinate with each other, and according to the proposed MTA algorithm, move to form another triangle to cover another part of the field. A MSN(or MSNs) can enter the sleep mode to conserve energy as the new triangle is being formed. The proposed algorithm enables the MSNs to minimize their (total and individual) travel distance or time needed to provide a full coverage of the field. Note that since the MSNs spent a constant amount of time performing sensing or sending beacon signals (depending on the application) while they are positioned as a triangle, the energy consumed by the MSNs is proportional to the total distance or time taken to complete the traversal. Accordingly, the proposed algorithm and its performance bound are also useful in minimizing and bounding the energy consumption of the MSNs.

This paper is organized as follows. In Section II, we formulate the problem. The proposed mobile traversing algorithm (MTA) is presented in Section III. In Section IV, bounds on the total distance and time required to complete the traversal process is derived. The minimum and maximum distance travelled by any individual MSN to complete the traversal is determined in Section V. Section VI concludes the paper.

II. Problem Formulation

A rectangular sensor field of length *L* and width *W* is divided into smaller equilateral triangles of sides *r*, where *r* is the radio range of the MSNs and $r \ll L, W$ as shown in Fig. 1. The stationary sensors in each triangular region reside at the intersection of the radio range of all three MSNs. The total number of such triangles is $(\frac{2(L+r)}{r} - 1) \times (\lceil \frac{2W}{r\sqrt{3}} \rceil)$, where $(\frac{2(L+r)}{r} - 1)$ and $(\lceil \frac{2W}{r\sqrt{3}} \rceil)$ are the number of triangles per row and column respectively and derived as the ratio of the area of a rectangular row or column to the area of a triangle. The additional coverage of $0.5r$ on either side of the rectangle ensures that the SSNs at the boundary fall within the radio range of all the MSNs. Finally, the extra triangle produced due to the extension of the row needs to be discarded. In order to identify a triangluar region, an index (*X*,*Y*) for X= $0 \ldots X_{max}$ and Y= $0 \ldots Y_{max}$, is assigned to a triangle where, X_{max} and Y_{max} represent the maximum row and column index and have values $\lceil \frac{2W}{r\sqrt{3}} \rceil - 1$ and $\frac{2(L+r)}{r} - 2$ respectively. Additonally, the shape of a triangle is identified by *formation* F_0 or F_1 where, F_1 has a vertex pointed upwards and F_0 is a mirror image of F_1 (Fig. 1).

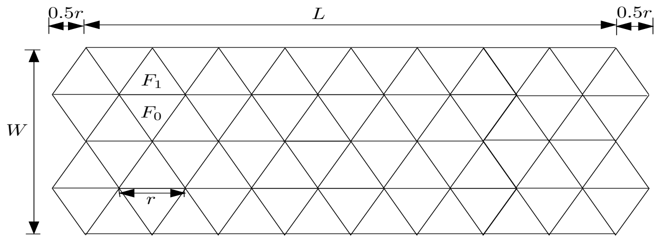


Fig. 1. Segmentation of the field into equilateral triangles

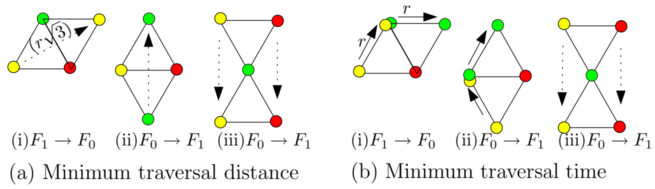


Fig. 2. Traversal between formations

The triangulation-based coverage problem we consider in this paper is as follows. For a random initial triangular placement of three MSNs, how to derive a mobile traversal algorithm (MTA) that minimizes the total distance and time taken by the MSNs to visit all the triangles in the field. Additionally, how to determine the individual distance travelled by each MSN. To ensure fairness, no single MSN should travel a significantly more distance compare to the others.

III. Mobile Traversing Algorithm

Starting at any initial index, the mobile traversing algorithm (MTA) splits the sensor field horizontally into two sub-rectangles, termed as the $topblock$ and the $bottomblock$. We assume that the starting index (X,Y) of the MSNs is known as it can be computed with respect to the coordinates of the rectangular field since the MSNs are assumed to be location-aware themselves. The MSNs may enter the top block and visit all the triangles in the block before moving to the bottom block and the vice versa. For an initial placement (X,Y) of the MSNs, the split can yield *Case (i)* a top block of $(0,0),(X,Y_{max})$ and a bottom block of $(X+1,0),(X_{max},Y_{max})$ or *Case (ii)* a top block of $(0,0),(X-1,Y_{max})$ and a bottom block of $(X,0),(X_{max},Y_{max})$. A single block is possible if X is either 0 or X_{max}.

Within a block, the MSNs initially move either towards the left or the right until an edge is reached. A row can be traversed two ways such that, either the distance or the time required to complete a move is minimized. The minimum distance travelled by the MSNs to move between formations F_0 and F_1 and the vice versa along a row is a diagonal move of distance $r\sqrt{3}$ by one of the MSNs as shown in Fig. 2(a)(i). In such a move, only the required MSN needs to move while the other two remain stationary. The MSN that makes the move is selected through collaboration between the MSNs. The MSNs select the one that has not participated in the last two moves and whose movement will produce a new triangular formation towards the desired traversal direction. On the other hand, to minimize the time required to complete a move across a row, two of the MSNs can simultaneously move in time r, along the sides of the triangle to complete the move (Fig. 2(b)(i)). In both cases, the stationary MSNs (or MSN) can enter the sleep mode for the duration of the transition phase (proportional to either $r\sqrt{3}$ or r) between formations to conserve energy.

At the edge, the MSNs move up or down along the edge until they reach either the first or the last row. The MSNs then traverse a row at a time until the end-point of the block is reached. For movements along a column, a downward movement from F_1 to F_0 or an upward movement from F_0 to F_1 (Fig. 2(a)(ii)) requires only one of the MSNs to travel a distance of $r\sqrt{3}$. On the other hand, such moves can be completed in time r if two of the MSNs move in parallel along the sides of the triangle (Fig. 2(b)(ii)). However, a downward movement from F_0 to F_1 (Fig. 2(a,b)(iii)) or an upward movement from F_1 to F_0 require two of the MSNs to each move a distance of $r\sqrt{3}$ in parallel to cover a new triangular region. In such a case, the minimum time needed to complete the move is $r\sqrt{3}$. After every two consecutive moves across a column, each MSN has travelled exactly a distance of $r\sqrt{3}$. The rotation among themselves for moves across a row and a column prevent excessive burden on any single MSN.

At the completion of the first block, the MSNs enter the second block and traverse it in a similar manner. The stop or end point of traversal for the MSNs in a block is determined from the number of rows and traversal direction in the block. In a block of even rows, based on its initial traversal direction, the MSNs either stop at the right or the left index horizontally adjacent to its starting point in the block. On the other hand, in a block of odd rows, an initial traversal towards the right or left leads the MSNs to stop at either the first or the last column of the starting row respectively. Fig.3 illustrates the proposed MTA for a starting index of (3,4) with an initial movement towards the left and the top block is traversed first (S_1 and S_2 represent the starting points and E_1 and E_2 the end-points of the top and the bottom block respectively).

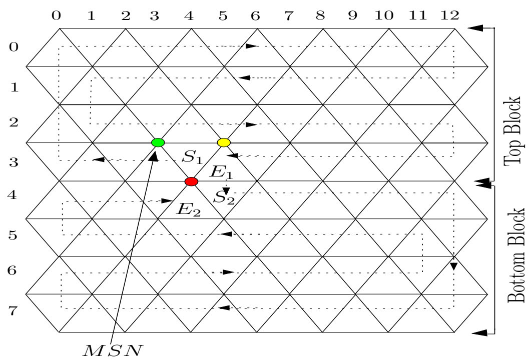


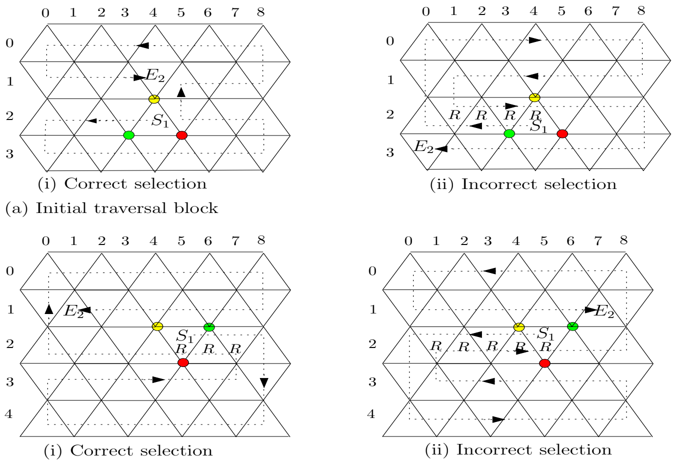
Fig. 3. An illustration of the MAT algorithm

A. Minimizing Revisits

In some cases, a triangle may be visited by the MSNs for more than once, resulting in "revisits" which translate to overhead in terms of additional travel distance and time by the MSNs. While revisits cannot be eliminated due to random initial deployment position (e.g., when the number of rows and columns are both odd, and either X or Y but not both are odd), the proposed MTA aims to reduce the additional overhead. This subsection discusses our strategy to reduce the overhead. Note that, owing to the fact that some vertical movement e.g., those shown in Fig. 2(a) and (b)(iii), result in more overhead than horizontal movement, it is not necessarily true that in case of inevitable revisits, minimizing the number of revisits will always minimize the overhead. For this reason, we consider below only row-wise movements described earlier in this section, and aim to reduce the number of revisits, instead of considering more elaborate approaches that may reduce the revisits by using more vertical movement.

Upon deployment, the MSNs determine the initial traversal direction and block based on the starting index. The initial traversal direction or block is determined according to X_{max} and governs the number of triangles re-visited by the MSNs. If X_{max} is odd then both the blocks will constitute an even number of rows only if the initial traversal block is chosen correctly. As an illustration, in Fig. 3, had the MSNs chosen to traverse the bottom block first then both the blocks would constitute of an odd number of rows. In a block of even rows, none of the triangles are revisited irrespective of the initial traversal direction because the MSNs leave and re-enter the starting row from opposite ends as depicted in Fig. 3. The top block will constitute of an even number of rows and be traversed first if the starting row index X is odd (since indexing starts with 0) whereas, the bottom block is of even rows and initially traversed if X is even. Fig. 4(a)(ii) illustrates that an incorrect selection of the initial traversal direction can cause almost half of the triangles in a row to be revisited (R denotes revisits).

On the other hand, if X_{max} is even then one of the blocks will always constitute of an odd number of rows regardless of which block is traversed first. In a block of odd rows, the MSNs leave and re-enter the starting row from the same end causing possible revisits of triangles. Therefore, if the MSNs initially move towards the outermost column nearer to its starting column index in the block, then the number of such revisits will be minimized. In Fig. 4(b)(i,ii), the correct initial traversal direction reduces the number of revisited triangles by almost half. Note that the correct selection of the initial traversal direction or block ensures that both the blocks do not constitute an odd number of rows.



(i) Correct selection (ii) Incorrect selection
(a) Initial traversal block
(i) Correct selection (ii) Incorrect selection
(b) Initial traversal direction

Fig. 4. Selection of the initial traversal block and direction

In the calculation of the total distance, each of the cost co-efficients has a value of $r\sqrt{3}$, the distance travelled per move (as shown in Fig. 2(a)). On the other hand, in the calculation of the total time, C_1 and C_3 have values r, the minimum time to complete a move across a row (Fig. 2(b)(i)). However, C_2 can have values r and $r\sqrt{3}$. C_2 is r for downward moves from F_1 to F_0 and upward moves from F_0 to F_1 (Fig. 2(b)(ii)) and is $r\sqrt{3}$ for other vertical moves (Fig. 2(b)(iii)). We term the moves in the former and the latter cases as N_{col_1} and N_{col_2} moves respectively. Therefore, the traversal cost across the columns for the total distance and time are $r\sqrt{3}(N_{col_1} + 2N_{col_2})$ and $rN_{col_1} + r\sqrt{3}N_{col_2}$ respectively. For the total distance, N_{col_2} is doubled because each of the two MSNs travel a distance of $r\sqrt{3}$ to complete the move. In the following subsections, we derive N_{row},N_{col} and N_{rev}.

A. Computation of N_{row}

N_{row} is computed as the sum of moves across the rows. The total number of rows is $X_{max} + 1$ since indexing starts with 0. The number of moves across the first and the last row is Y_{max}, the number of triangles per row minus the one from where the move has started in the row. However, for the rows in between, $Y_{max} - 1$ moves are made across a row since the outermost index has already been traversed by the MSNs while reaching the first or the last row. Thus, N_{row} is computed according to equation 2.

$$N_{row} \quad = 2Y_{max} + (X_{max} - 1)(Y_{max} - 1) \tag{2}$$

B. Computation of N_{col}

In MTA, the MSNs can change rows at the two outermost columns on either sides of the rectangular field. An exception may only occur when the MSNs switch blocks. In a block, the moves across the columns can be determined from the *checking indexes*. A checking index (CI) is an index such as (X_i, Y_j) from where the traversal across a column starts. The formation F_0 or F_1 at a CI can determine the number of such moves. In a block, the CI are the pair of indexes diagonally

IV. Determining the Total Distance and Time

The cost function, C_{tot}, for the minimum total distance and time taken by the MSNs to provide a full coverage of the field is presented in equation 1. C_{tot} is the sum of the cost of traversing the rows and columns and possible revisits.

$$C_{tot} \quad = C_1 N_{row} + C_2 N_{col} + C_3 N_{rev} \tag{1}$$

In equation 1, N_{row} and N_{col} represent the total number of moves across the rows and columns respectively, and N_{rev} accounts for the number of possible revisited triangles across a row. Additionally, C_1, C_2 and C_3 are the cost co-efficients.

opposite to each other and are chosen according to the initial traversal direction and position of the block. Assuming X_s to be the starting traversal row in a block, the pair of CI for the top block can be either $(X_s, 0)$ and $(0, Y_{max})$ or (X_s, Y_{max}) and $(0, 0)$ for an initial traversal towards the left or right respectively. On the other hand, in the bottom block, $(X_s, 0)$ and (X_{max}, Y_{max}) or (X_s, Y_{max}) and $(X_{max}, 0)$ are the pair of CI for an initial traversal towards the left or right respectively. X_s corresponds to X if the block is the first one traversed otherwise, it is $X-1$ or $X+1$ for the top or bottom block respectively.

In a block, the formations at the pair of CI can be determined from the formation at the starting index (X_s, Y_s) of the block. $(X_s, 0)$ has the same formation as (X_s, Y_s) if the MSNs require an even number of row-wise moves to reach the first column. Otherwise, it has the formation opposite to (X_s, Y_s). Similar terms apply to (X_s, Y_{max}) when the MSNs traverse towards the last column of the rectangular field. The formations at the other four pairs of CI can be determined with respect to the first two. $(0, 0)$ and $(0, Y_{max})$ have the same formation as $(X_s, 0)$ and (X_s, Y_{max}) respectively, if the MSNs, starting at row X_s, can reach the first row with an even number of column-wise moves. Under similar terms, the formations at (X_{max}, Y_{max}) and $(X_{max}, 0)$ can be determined when the MSNs traverse towards the last row of the block. We term $(X_s, 0)$ or (X_s, Y_{max}) as the first checking index (FCI) and the other as the second checking index (SCI) of a block.

1) Number of moves from FCI: In a block, the traversal across the column starts at the FCI. From FCI, the MSNs move either upward or downward (across either the first or last column) until either the first or the last row is reached. In the top block, a total of X such moves are made to reach the first row of the block. Whereas, $X_{max} - X$ moves are needed to reach the last row in the bottom block. As illustrated in Fig.2(a,b)(iii), every two consecutive moves across a column accounts for a N_{col_1} and a N_{col_2} move. Therefore, if an even number of moves are made from FCI then N_{col_1} and N_{col_2} will each be responsible for either $\frac{X}{2}$ or $\frac{X_{max}-X}{2}$ moves depending on the traversal block. Otherwise, based on the formation of FCI and the traversal direction across the column, either N_{col_1} or N_{col_2} will have an additional move. If FCI has formation F_0 and the MSNs move upward, then N_{col_1} will have $\lceil \frac{X}{2} \rceil$ moves, compare to the $\lfloor \frac{X}{2} \rfloor$ moves made by N_{col_2}. The contrary holds if the formation at FCI is F_1. For a downward move from FCI, the computation is reversed with the number of moves for N_{col_1} and N_{col_2} to be $\lceil \frac{X_{max}-X}{2} \rceil$ or $\lfloor \frac{X_{max}-X}{2} \rfloor$.

As the MSNs traverse back towards the starting row in a block (from either the first or last row), they change rows alternatingly at *Case(i)* the column farthest from FCI (i.e. the other outermost column) and *Case(ii)* the column adjacent to FCI. The moves along the former and latter cases can be determined from the formation at SCI and FCI respectively. In the latter case, the traversal across the column starts from the second outermost row and the total number of such moves is either $\lfloor \frac{X}{2} \rfloor$ or $\lfloor \frac{X_{max}-X}{2} \rfloor$. If the traversed block has an even number of rows then the MSNs re-enter the starting row from the column not adjacent to FCI. This implies that the MSNs do not move from the formation at FCI to its complement at the adjacent column (note the index horizontally adjacent to FCI has the formation opposite to FCI). Therefore, the MSNs must only change formation (to the formation at FCI) at alternate rows. If FCI has formation F_0 and the top block is being traversed then N_{col_1} corresponds to $\lfloor \frac{X}{2} \rfloor$. On the other hand, if the bottom block is being traversed, then N_{col_2} corresponds to $\lfloor \frac{X_{max}-X}{2} \rfloor$. The opposite holds if FCI has formation F_1. On the contrary, in a block of odd rows, the MSNs re-enter the starting row through the column adjacent to FCI. In such case, MSNs move from the formation at FCI to its complement at the adjacent column. Thus, if FCI has formation F_0 then N_{col_2} and N_{col_1} correspond to $\lfloor \frac{X}{2} \rfloor$ and $\lfloor \frac{X_{max}-X}{2} \rfloor$ for the top and bottom block respectively.

2) Number of moves from SCI: As mentioned previously, the formation at SCI can determine the number of moves along the outermost column visited by the MSNs as they traverse back towards the starting row of the block. In this case, the traversal across the column starts from either the first or the last row of the block. Thus, a total of $\lceil \frac{X}{2} \rceil$ or $\lceil \frac{X_{max}-X}{2} \rceil$ moves are possible. Like the moves across the column adjacent to FCI, N_{col_1} and N_{col_2} can be similarly computed according to the size and position of the traversed block and the formation at SCI.

The computation of N_{col_1} and N_{col_2} under the different traversal scenarios is presented in Procedure 1. In Procedure 1, X_{s_1} and X_{s_2} represent the starting row index for the top and bottom block respectively. If the top block is traversed first then X_{s_1} corresponds to X and X_{s_2} is X+1. Otherwise, X_{s_2} and X_{s_1} correspond to X and $X-1$ respectively.

C. Computation of N_{rev}

As explained in section III-A, in order to minimize the number of revisited triangles in a block of odd rows, the MSNs initially traverse towards the outermost column closer to its starting index. Thus, for the starting column index Y_s in a block, N_{rev} is computed according to equation 3. The revisits along the outermost two columns are ignored as they have been considered in N_{col}.

$$N_{rev} = Minimum(Y_s - 1, Y_{max} - Y_s - 1) \tag{3}$$

D. Bounds for the total distance and time

In the derivation of the bounds, N_{row} remains constant for both the lower and upper bounds since a move across a row is completed within a contant time or travelling distance. On the other hand, N_{rev} is 0 for the lower bound and an upper bound is attained by setting Y_s in equation 3 to $\frac{Y_{max}}{2}$ to attain an upper bound of $\frac{Y_{max}}{2} - 1$.

In order to derive the bounds on N_{col}, we first determine the total number of moves across the columns. If the top block is initially traversed then N_{col} for the first and second block are $\lceil \frac{X}{2} \rceil + \lfloor \frac{X}{2} \rfloor + \lceil \frac{X}{2} \rceil + \lfloor \frac{X}{2} \rfloor$ or $2X$ and $\lceil \frac{X_{max}-X-1}{2} \rceil + \lfloor \frac{X_{max}-X-1}{2} \rfloor + \lceil \frac{X_{max}-X-1}{2} \rceil + \lfloor \frac{X_{max}-X-1}{2} \rfloor$ or $2(X_{max} - X - 1)$ respectively. Otherwise, N_{col} for the first and second block is $2(X_{max}-X)$ and $2(X-1)$ respectively. In either case, including the transition move between blocks, the summation

Procedure 1 Computation of N_{col}

if $FCI == F_0$ **then**
  if $TopBlock$ is $Even$ and $BottomBlock$ is $Even$ **then**
    $N_{col_1} \leftarrow X_{s_1} + \lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
    $N_{col_2} \leftarrow \lfloor \frac{X_{s_1}}{2} \rfloor + X_{max} - X_{s_2}$
  else if $TopBlock$ is $Even$ and $BottomBlock$ is Odd **then**
    $N_{col_1} \leftarrow X_{s_1} + 2\lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
    $N_{col_2} \leftarrow \lfloor \frac{X_{s_1}}{2} \rfloor + \lceil \frac{X_{max}-X_{s_2}}{2} \rceil$
  else if $TopBlock$ is Odd and $BottomBlock$ is $Even$ **then**
    $N_{col_1} \leftarrow \lceil \frac{X_{s_1}}{2} \rceil + \lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
    $N_{col_2} \leftarrow 2\lfloor \frac{X_{s_1}}{2} \rfloor + X_{max} - X_{s_2}$
else
  if $TopBlock$ is $Even$ and $BottomBlock$ is $Even$ **then**
    $N_{col_1} \leftarrow \lfloor \frac{X_{s_1}}{2} \rfloor + X_{max} - X_{s_2}$
    $N_{col_2} \leftarrow X_{s_1} + \lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
  else if $TopBlock$ is $Even$ and $BottomBlock$ is Odd **then**
    $N_{col_1} \leftarrow \lfloor \frac{X_{s_1}}{2} \rfloor + \lceil \frac{X_{max}-X_{s_2}}{2} \rceil$
    $N_{col_2} \leftarrow X_{s_1} + 2\lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
  else if $TopBlock$ is Odd and $BottomBlock$ is $Even$ **then**
    $N_{col_1} \leftarrow 2\lfloor \frac{X_{s_1}}{2} \rfloor + X_{max} - X_{s_2}$
    $N_{col_2} \leftarrow \lceil \frac{X_{s_1}}{2} \rceil + \lfloor \frac{X_{max}-X_{s_2}}{2} \rfloor$
if $SCI == F_0$ **then**
  $N_{col_1} \leftarrow N_{col_1} + \lceil \frac{X_{max}-X_{s_2}}{2} \rceil$
  $N_{col_2} \leftarrow N_{col_2} + \lceil \frac{X_{s_1}}{2} \rceil$
else
  $N_{col_1} \leftarrow N_{col_1} + \lceil \frac{X_{s_1}}{2} \rceil$
  $N_{col_2} \leftarrow N_{col_2} + \lceil \frac{X_{max}-X_{s_2}}{2} \rceil$

of N_{col} for both the blocks is $2X_{max} - 1$. A lower bound on N_{col} is attained when N_{col_2} is minimized and N_{col_1} is maximized. N_{col_1} is maximized if the moves from SCI and the column adjacent to FCI in both the blocks are N_{col_1} moves. Then, including the transition move between blocks, N_{col_1} is $\lceil \frac{X}{2} \rceil + \lfloor \frac{X}{2} \rfloor + \lceil \frac{X_{max}-X}{2} \rceil + \lfloor \frac{X_{max}-X}{2} \rfloor$ or X_{max}. If the top block is initially traversed then the minimum N_{col_2} move from the FCI is $\lfloor \frac{X}{2} \rfloor + \lfloor \frac{X_{max}-X-1}{2} \rfloor$. If X and X_{max} are odd, then N_{col_2} has its minimum value of $\frac{X-1}{2} + \frac{X_{max}-X-2}{2}$ or $\frac{X_{max}-3}{2}$. In such case, N_{col_1} will have one extra move in each block than N_{col_2} for a total of $\frac{X_{max}+1}{2}$ moves in the two blocks. A similar bound is achieved if the bottom block is traversed first. Therefore, N_{col_1} and N_{col_2} have a lower bound of $\frac{X_{max}-3}{2}$ and an upper bound of $\frac{3X_{max}+1}{2}$ respectively.

By substituting N_{row}, N_{col} and N_{rev} in equation 1 with the above bounds, we obtain a lower and an upper bound on the total distance and time. In the following theorems, X_{max} and Y_{max} are replaced by $\lceil \frac{2W}{r\sqrt{3}} \rceil - 1$ and $\frac{2(L+r)}{r} - 2$ respectively, to obtain the bounds interms of the dimension of the sensor field and r.

Theorem 1: In a rectangular field of length L and width W, tiled into equilateral triangles of sides r, three MSNs require a minimum total travelling distance of $2\sqrt{3}L\lceil \frac{2W}{r\sqrt{3}} \rceil + \frac{3r\sqrt{3}}{2}(\lceil \frac{2W}{r\sqrt{3}} \rceil - 2)$ or a total time of $r(\lceil \frac{2W}{r\sqrt{3}} \rceil (\frac{2L}{r} + \frac{1}{2}) + 1) + \frac{r\sqrt{3}}{2}(\lceil \frac{2W}{r\sqrt{3}} \rceil - 4)$ to provide a triangulation-based coverage of the field.

Theorem 2: In a rectangular field of length L and width W, tiled into equilateral triangles of sides r, three MSNs require a maximum total travelling distance of $2\sqrt{3}L(\lceil \frac{2W}{r\sqrt{3}} \rceil + \frac{1}{2}) + \frac{r\sqrt{3}}{2}(5\lceil \frac{2W}{r\sqrt{3}} \rceil - 6)$ or a total time of $r(\lceil \frac{2W}{r\sqrt{3}} \rceil (\frac{2L}{r} - \frac{1}{2}) - 1) + L + \frac{r\sqrt{3}}{2}(3\lceil \frac{2W}{r\sqrt{3}} \rceil - 2)$ to provide a triangulation-based coverage of the field.

The bounds are verified by simulation as follows. In a sensor field of dimension 5000×2000 and r set to 50, 100 random initial starting index and formation of the MSNs are chosen. For each starting index, the actual total distance and time are calculated according to equation 1 and the bounds are determined according to the above theorems. The results are presented in Fig. 5 and Fig. 6 to justify the tightness of the computed bound. In the following section, the individual traversal distance of a MSN is computed from the tight bounds on the total distance.

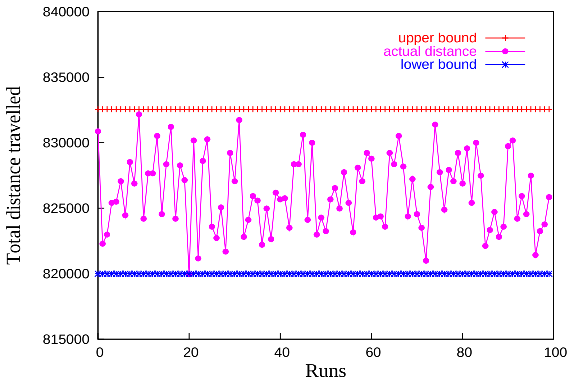


Fig. 5. The total distance travelled

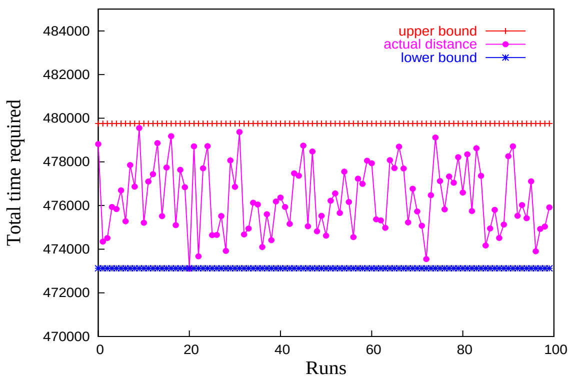


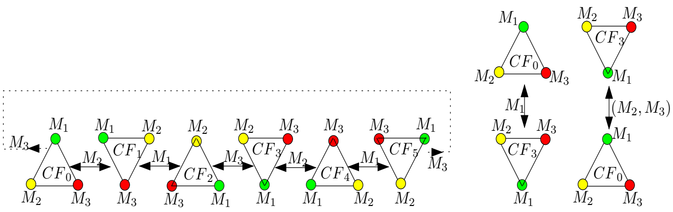
Fig. 6. The total distance travelled

V. Determining the Bounds on Individual Traversal Distance

In this section, we determine the minimum and maximum distance travelled by a MSN to complete the traversal process. Since a constant time is spent as a trianglular formation, the energy dissipated by an MSN is proportional to the distance it travels. In the following subsections, we determine the best or *most fair* case and the worst or *least fair* case of movements by the MSNs across the rows and the columns. In the most fair case, the difference in the distance travelled by any two MSNs is the minimum. The contrary holds for the least fair case.

A. Moves across the rows

The traversal pattern of the three MSNs (M_1,M_2 and M_3) for moves across a row is presented in Fig. 7(a). According to the pattern, only six configurations (CF_0-CF_5) can represent the possible positioning of the MSNs after a move. Also note that, each MSN has moved once in every three consecutive moves. Thus, fairness is achieveable if the number of moves across a row is a factor of 3. In other cases, unfairness results in one or two of the MSNs to make an extra move than the third one. Since $X_{max} - 1$ inner rows are present (equation 2), in the most fair case, each of the MSNs is responsible for a total of exactly $(X_{max} - 1)(\frac{Y_{max}-1}{3})$ moves across all the inner rows. However, the extra move per outermost row requires one of the MSNs to make two more moves than the other two for a total of $2(\frac{(Y_{max}-1}{3} + 1)$ moves. The other two make $\frac{2(Y_{max}-1)}{3}$ moves. Therefore, under the most fair case, one of the MSNs makes only two extra moves than the other two.



(a) Across the row (b) Across the column

Fig. 7. Traversal patterns of the MSNs

On the other hand, in the least fair case (assuming the number of moves across an inner row not be a factor of 3), each MSN makes a total of atleast $(X_{max}-1)(\frac{Y_{max}-2}{3})$ moves and two of them are required to make an extra move per row. For the outermost rows, an additional move per row allows each to participate in exactly $\frac{2Y_{max}}{3}$ moves. Therefore, in the least fair case, two of the MSNs each make $X_{max} - 1$ more moves than the third one. For the moves across the rows, the least fair case examplifies the minimum and maximum number of moves made by any single MSN.

B. Moves across the columns

The traversal pattern of the MSNs in Fig. 7(b) suggests that the MSNs toggle between only two configurations as they move across the column. Furthermore, each MSN completes a move in every two consecutive moves across a column. Since a lower and an upper bound is possible for N_{col}, the fairness degree in each is examined separately. A lower bound on N_{col} is achieved when a single MSN (i.e. N_{col_1} moves) makes X_{max} moves from SCI and the column adjacent to FCI in both the blocks. In the most fair case, the moves are divided equally among the MSNs ($\frac{X_{max}}{3}$ per MSN) whereas, a single MSN may be responsible for all X_{max} moves in the least fair case. From FCI, a lower bound is achieved when the number of N_{col_1} moves per block is one more than N_{col_2} (section IV-D). In the most fair case, two different MSNs make the extra move in each block such that, each account for a total of $\frac{X_{max}-1}{2}$ moves. The third MSN, in this case, is responsible for $\frac{X_{max}-3}{2}$ moves. However, in the least fair case, one of the MSNs makes the additional move in both the blocks for a total of $\frac{X_{max}+1}{2}$ moves. In comparison, the other two MSNs make $\frac{X_{max}-3}{2}$ moves each. To summarize, under the most fair case, two of the MSNs make one extra move than the third whereas, a single MSN may complete $X_{max} + 2$ more moves than the other two in the least fair case. Thus, the minimum number of moves made by a MSN to complete the traversal across the columns is $\frac{X_{max}-3}{2}$ (in the least fair case).

An upper bound on N_{col} is possible if the moves from SCI and the column adjacent to FCI are both N_{col_2} moves. In the most fair case, a different MSN participates in all the moves for the two blocks and a different pair must traverse each column of a block. Then, the difference in number of moves between any two MSN is $\frac{X_{max}}{2}$. In contrast, the least fair case is possible if the same pair of MSNs make all X_{max} moves. An upper bound from FCI is achieved if the number of N_{col_2} moves per block is one more than N_{col_1}. Then, in the most fair case, two different pairs of MSNs can make the extra move in each block. This allows two of the MSNs to each make $\frac{X_{max}-1}{2}$ moves whereas, the third one (that participated in both the extra moves) account for $\frac{X_{max}+1}{2}$ moves. In the least fair case, the same pair of MSNs may make additional moves in both the blocks for a total of $\frac{X_{max}+1}{2}$ moves. The third MSN, in this case, makes $\frac{X_{max}-3}{2}$ moves. To summarize, under the most fair case, two of the MSNs are responsible for $\frac{X_{max}}{2} + 2$ and $\frac{X_{max}-X}{2} + 2$ more moves than the third one whereas, the number of such additional move for each is $X_{max}+2$ in the least fair case. Then, the maximum number of moves required by any MSN to complete the traversal across the columns is $X_{max} + \frac{X_{max}+1}{2}$ or $\frac{3X_{max}+1}{2}$ moves in the least fair case.

C. Determining the Bounds

In order to complete the bound on the maximum individual traversal distance, the least fair case for the upper bound on N_{rev} is considered. In such case, two of the MSNs make an extra move across the row for a total of $\frac{Y_{max}}{6}$ moves. We derive the theorem for the bounds on individual traversal distance as the sum of the minimum and maximum distance traversed by any MSN across the rows, columns and due to revisits.

Theorem 3: In a rectangular field of length L and width W, tiled into equilateral triangles of sides r, it takes any of the three MSNs a minimum total travelling distance of $2\sqrt{3}L\lceil\frac{2W}{r\sqrt{3}}\rceil - \frac{r\sqrt{3}}{2}(3\lceil\frac{2W}{r\sqrt{3}}\rceil + 4)$ and a maximum of $2\sqrt{3}L(\lceil\frac{2W}{r\sqrt{3}}\rceil + \frac{1}{6}) + \frac{r\sqrt{3}}{2}(5\lceil\frac{2W}{r\sqrt{3}}\rceil - 6)$ to provide a complete coverage of the field.

VI. Conclusion

We have presented a mobile traversal algorithm for a triangulation-based coverage of a field. MTA has two distinctive advantages. First, starting at a random initial starting index, MTA minimizes the total traversing distance or time required by three MSNs to cover the field. Second, the bounds on the total and individual traversing distance and time can be computed *a priori* with respect to the number and dimension of possible triangular formations. Furthermore, in MTA, a MSN (or MSNs) enter the sleep mode during the transition phase between formations to conserve energy. Due to the limited number of MSNs, MTA incurs variable latency in coverage of each region of the field. However, sub-sectioning the field into smaller regions and deploying more MSNs would reduce such latency.

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